ECS 452: Quiz 1

Instructions

Name		ID

1. Write each of the following quantities in the form X.XXX (possibly with the help of your calculator).

   a. $-\log_2(0.125) = -\log_2\left(\frac{1}{8}\right) = -\log_2 2^{-3} = -(-3) = 3 \approx 3.000$

   b. $-\log_2(0.1) \approx 3.322$

   c. $-(0.2)\log_2(0.2) - (0.8)\log_2(0.8) \approx 0.722$
   $\underbrace{}_{0.4644} \quad \underbrace{}_{0.2575}$

2. Consider a random variable X having four possible values. Their probabilities are

$$1/8, \; 5/24, \; 7/24, \; 3/8.$$

   a. Find the **expected codeword length** (per symbol) when **Huffman coding** is used (without extension) to encode an i.i.d sequence generated by this random variable.

Recipe:
Repeatedly group the two least likely symbol).

$p_X(x)$

$1/8 = 3/24$ — $8/24$
$5/24$ —
$7/24$ — $15/24$
$3/8 = 9/24$ — $24/24 = 1$
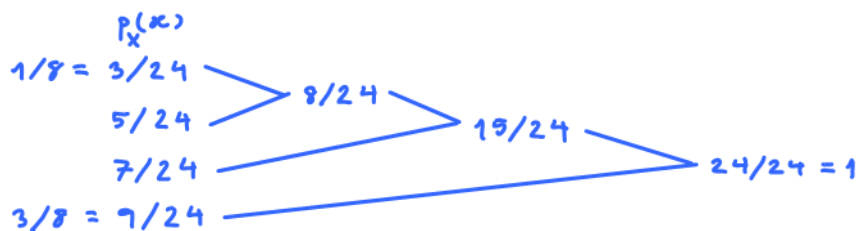
$\ell(x)$
3
3
2
1

Note that we can find $\ell(x)$ without writing down $c(x)$ first.

$$\mathbb{E}[\ell(X)] = 3 \times \frac{3}{24} + 3 \times \frac{5}{24} + 2 \times \frac{7}{24} + 1 \times \frac{9}{24} = \frac{9+15+14+9}{24} = \frac{47}{24} \approx 1.9583$$

   b. Find the **entropy** (per symbol) of this random variable.

$$H(X) = -\sum_x p_X(x) \log_2 p_X(x)$$

$$= -\frac{3}{24}\log_2\frac{3}{24} - \frac{5}{24}\log_2\frac{5}{24} - \frac{7}{24}\log_2\frac{7}{24} - \frac{9}{24}\log_2\frac{9}{24}$$

$$\approx 0.3750 + 0.4715 + 0.5185 + 0.5306$$

$$= 1.8956$$