

ECS 452: Quiz 1

Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group.
3. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID

1. Write each of the following quantities in the form X.XXX (possibly with the help of your calculator).

a. $-\log_2(0.125) = -\log_2\left(\frac{1}{8}\right) = -\log_2 2^{-3} = -(-3) = 3 \approx 3.000$

b. $-\log_2(0.1) \approx 3.322$

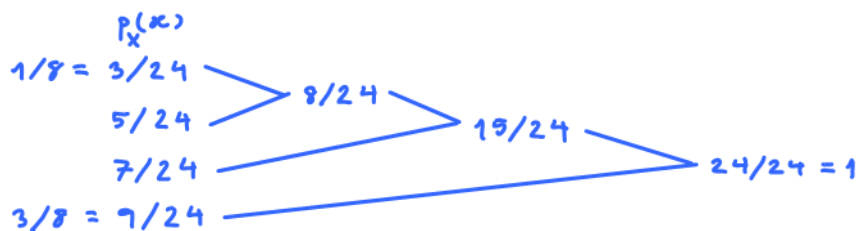
c. $\underbrace{-(0.2)\log_2(0.2)}_{0.4644} - \underbrace{(0.8)\log_2(0.8)}_{0.2575} \approx 0.722$

2. Consider a random variable X having four possible values. Their probabilities are

$$1/8, 5/24, 7/24, 3/8.$$

a. Find the **expected codeword length** (per symbol) when **Huffman coding** is used (without extension) to encode an i.i.d sequence generated by this random variable.

Recipe:
Repeatedly group the two least likely symbols.



$$l(x)$$

3
3
2
1

Note that we can find $l(x)$ without writing down $c(x)$ first.

$$E[l(X)] = 3 \times \frac{3}{24} + 3 \times \frac{5}{24} + 2 \times \frac{7}{24} + 1 \times \frac{9}{24} = \frac{9+15+14+9}{24} = \frac{47}{24} \approx 1.9583$$

b. Find the **entropy** (per symbol) of this random variable.

$$\begin{aligned}
 H(X) &= -\sum_x P_X(x) \log_2 P_X(x) \\
 &= -\frac{3}{24} \log_2 \frac{3}{24} - \frac{5}{24} \log_2 \frac{5}{24} - \frac{7}{24} \log_2 \frac{7}{24} - \frac{9}{24} \log_2 \frac{9}{24} \\
 &\approx 0.3750 + 0.4715 + 0.5185 + 0.5306 \\
 &= 1.8956
 \end{aligned}$$